ONE-VALUED LOGIC

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Consider the following argument: Logic must be *two-valued*; for if language is to be capable of giving information—of "making a claim on reality"—it must be capable of giving false information, making a false claim. Consequently, what is asserted it must also be possible to deny. If there could be no falsehood there could be no truth either, and hence no communication.

This argument is confused.

As an exercise in logical poverty it is of interest to consider a language which might reasonably be called "one-valued". In a formal sense it is an extremely trivial language. It is, however, capable of representing the logical structure of various rudimentary languages that have been considered by writers concerned with Logic's less formal features: Quine's "rabbit" or "gavagai" language in *Word and Object*; the language of "feature-placing sentences" in Strawson's *Individuals*; and Wittgenstein's "slab" language. It also has some connections with features of the logic of questions, and the logic of ejaculations.

Let us first, however, consider it formally as an uninterpreted system in the manner of the logic books. That is, let us introduce our symbols and define our formulae and set down the axioms and rules of inference to be used in deriving theorems from them.

Undefined symbols. Variables 'p', 'q', 'r', etc. will be called "statement-variables", subject to the proviso, more important here than normally, that the name is arbitrary and that they do not necessarily represent statements properly so-called. We also have two operators 'K' and 'A', to be called "conjunction" and "disjunction" respectively. We use Polish symbol-order and hence define *formulae* by the rules:

(1) $p, q, r, \ldots$ are formulae.

(2) Whenever α and β are formulae, so are $K\alpha\beta$ and $A\alpha\beta$.

Axioms. There are no axioms.

Rules of inference. There are, however, more rules of inference than usual.

(i) In the first place we have the usual rule of substitution of a formula for a variable in any valid formula.

(ii) Next we have a number of specific rules of deduction which may be summed up in the schemata

$$\alpha, \beta \rightarrow K\alpha\beta \rightarrow K\beta\alpha \rightarrow \alpha \rightarrow A\alpha\beta \rightarrow A\beta\alpha$$

and

$$A\alpha\alpha \rightarrow \alpha.$$

(iii) And finally we have the rule of substitution of equivalents. By 'equivalents' we shall mean formulae which are interdeducible by rule (ii) as just given.

The axioms and rules of inference together define the *theorems* of the language. A theorem is, as usual, any formula which is either an axiom or deducible from the axioms in accordance with the rules of inference. However, since there are no axioms there cannot be any formulae deducible from axioms, and it follows that there are no theorems.

What is the point of a system without axioms and theorems? The first thing to be said is that there could not very well be any axioms or theorems in this system so long as the operators 'K' and 'A' represent something like what we usually call "conjunction" and "disjunction"; for in an ordinary two-valued logic we cannot express any axioms or theorems in terms of these operators alone. This can easily be proved by an argument from truth-values, since axioms and theorems must always have the value 'true', whereas 'K' and 'A' class-close on 'false'. So objection can be taken only on the grounds of triviality. We shall see that the triviality is formal only, in that the system is quite capable of serving as the logical skeleton of a possible language.

What is the point of rules of inference in a system which has no axioms and hence nothing for the rules to get to work on? It is only relative to a set of rules of inference that we can define 'theorem', and hence if we did not set out rules of inference it would not make sense for us to say that the system has no theorems. For that matter, it would hardly make sense for us to say, in this case, that there are no axioms. But why just *these* rules of inference? Well, we are partly interested in the system's counter-factual properties as well as its actual ones: it is not irrelevant to our considerations that *if* the system had had these and those axioms it would have had this and that theorem.

But there is another point about the rules of inference, and it involves the resolution of a misconception about what rules of inference are. The rules of inference currently in vogue in formal logic in fact fall into two classes. There are those which, like *modus ponens*, are of general validity as rules of inference in the classical sense; and there are those which are of use only in deducing theorems. The first sort guarantees the truth of conclusions inferred from true premisses: the second guarantees the necessity—that is, analyticity or theoremhood—of conclusions inferred from premisses which are necessary—that is, from axioms and other theorems. An example of the second sort of rule is the rule of substitution. From the tautology 'It is either raining or not raining' it is permissible to deduce, by substituting 'it is snowing' for 'it is raining', the further tautology 'It is either snowing or not snowing'; but it would be quite illegitimate to use substitution to infer from the mere *truth* of 'It is raining now outside' the *truth* of 'It is snowing now outside'.

The purpose of propositional logic is to exhibit or model the logical rules that hold within language; and, in particular, relations of inference or deducibility. But on account, perhaps, of the fact that it was developed mainly in connection with pure mathematics where, as the doctrine goes, anything that is true at all is necessarily true, modern propositional logic conflates or confuses the two kinds of inference rule. What makes this *appear* to be in order is that the system symbolizes within itself the concept of implication, whence it can plausibly (though inaccurately) be claimed that the inference of merely-true statements from other merely-true statements is duly allowed for.

If, however, the need is removed, within the system, for the deduction of theorems, it by no means follows that rules of inference are not needed at all; for it may still be necessary—in order, that is, to fulfil the primary purpose of formal logic—to indicate the logical relations that hold between the various formulae the system contains. Hence some of our rules of inference—those of classes (ii) and (iii)—have this additional function: they are rules, permitting in general the inference of true statements from true, that one would insist be satisfied in any interpretation of the system, independently of their roles in connection with (in this case non-existent) axioms or theorems or necessary truths.

It seems, incidentally, to be a feature of any rule of inference which yields true conclusions when applied to true premisses that it yields a necessarily true conclusion whenever it is applied to necessarily true premisses. (But I do not know how to prove this.)

Now let us prove the *consistency* of the system. The principal difficulty is that of deciding what we mean by 'consistency' in the present case. If we take it that the test of consistency is that no contradiction be derivable, then the present system seems certainly to satisfy it since nothing is derivable at all. But it is far from clear what would *count* as a contradiction. Nothing? If so, the statement that no contradiction is derivable is empty.

Consistency is sometimes defined as existence of at least one non-provable formula; or, equivalently, as non-provability of the formula 'p'. This definition is clearly applicable to the present system, and 'p' is in fact not provable. It is possible, of course, to doubt whether there would be "any inconsistency" in having all the formulae of the present system provable at once, including 'p'. However, crushing these doubts, let us accept the definition and the result.

Now, is the system *complete*? One would think that a system with no axioms was as incomplete as could possibly be. But actually, subject to the definition just given of 'consistency', we can prove that the present system is complete: in terms of the operators 'K' and 'A' it is not possible to add an independent axiom—not possible, that is, to have any axioms at all—without making the system inconsistent in the sense that all formulae would be theorems.

Thus, starting from any axiom, we could

(i) use substitution to put 'p' for any statement-variables that were not 'p' already; and

(ii) use the easily-proved equivalence of both $K\alpha\alpha$ and $A\alpha\alpha$ to α to reduce the resulting formula by substitution of equivalents until it was simply 'p'.

Thus 'p' would be a theorem and, by substitution, all formulae.

Now for interpretations.

Strawson (in *Individuals*[1], p. 212) says "If any facts . . . deserve to be called ultimate or atomic facts, it is the facts stated by those propositions which demonstratively indicate the incidence of a general feature". And (on p. 202) he speaks of "feature-placing statements", with the examples

Now it is raining.
Snow is falling.
There is coal here.
There is gold here.
There is water here.

These statements are quite unlike atomic sentences in the sense of Russell or Wittgenstein, because they do not contain or involve reference to any "re-identifiable particulars", such as "veins, grains, lumps or dumps of coal or flakes, falls, drifts or expanses of snow": just *coal* or *snow*, the general kind of stuff. Strawson further remarks that the sense of the word 'here', or a similar demonstrative word, seems to be essential, in that it must be possible over a period to state different facts with the same stock of words. One does this in saying the words over again at a significantly different time, or after having moved or pointed to a significantly different place.

We can now quite easily imagine a rudimentary language which contains statements like these but no others. The people who speak this language have a stock of (let us say) one-word elementary statements. When they say 'Queazle' they mean 'It is snowing'; and when they say 'Gavagai' they mean 'There is at least one rabbit about'. But we can now also imagine that they sometimes say '*K* queazle gavagai' to mean 'It is snowing and there is a rabbit about', or '*A* queazle gavagai' to mean 'Either it is snowing or there is a rabbit about or both'; and we can imagine that just very occasionally they say much longer things like '*A K* queazle gavagai *K* gavagai queazle' and that this is to be translated in a way which conforms to expectations. That is, the people who speak this language move about making statements which are comments on features of their environment, sometimes plain and sometimes conjunctive or disjunctive, or a mixture.

There would be logical relations in such a language since whenever someone said '*K* gavagai queazle' he would thereby imply, or implicitly also assert, the simpler statement 'Gavagai'. But we can quite easily imagine

[1] P. F. Strawson, *Individuals* (London, 1959).

that, although some of the comments were true and some false, and although the comments would give information, true or false, to anyone who heard them and who was not as well placed as the speaker to observe the relevant environment, there might be no provision in the language for *denying* the presence of rabbits or snow and no provision for expressing the existence of the logical relations.

This is put, however, in a form some people would consider naive and question-begging. How would you know, it will be asked, what these various statements mean ? How will you know that 'gavagai' refers to rabbits rather than rabbit-tails, rabbit-behaviour or rabbit-segments ? How will you even know that the utterance 'Gavagai' represents a *statement* as distinct from some kind of order or ejaculation, or even a question such as 'Is that a rabbit ?' ?

In connection with the last question let us return to the argument with which we started : If there could be no denial there could be no truth, and hence no communication. The argument—which we must answer—would be elaborated in the following form, following Wittgenstein (in, chiefly, *The Brown Book*) and Quine[2]. Imagine (they would say) that there is a tribe of people who go about making noises of the kind you describe in the kinds of situation you describe, and that you go along to observe them and to translate their language into our own on the basis of your observations. Your knowledge of the grammar of their language is put together from observation in the same way as that of the vocabulary. But under these circumstances there is nothing to determine their utterances as what we would call "statements" (or "propositions" or "indicatives", etc.). It is the possibility of denial that makes statements statements.

Thus Quine says (p. 29) :

> A rabbit scurries by, the native says 'Gavagai', and the linguist notes down the sentence 'Rabbit' (or 'Lo, a rabbit') as tentative translation, subject to testing in further cases. The linguist will at first refrain from putting words into his informant's mouth, if only for lack of words to put. When he can, though, the linguist has to supply native sentences for his informant's approval. . . . So we have the linguist asking 'Gavagai ?' in each of various stimulatory situations, and noting each time whether the native assents, dissents, or neither.

He goes on to consider the difficulties of recognising assent and dissent, but thinks we shall succeed here if we guess intelligently.

Quine's reason for thinking that we must get round to asking questions and eliciting assent or dissent is that he thinks that otherwise we would never be able to sort out the meanings of those terms which had references in common. Suppose that the language contains a word for 'Rabbit' and a word for 'Animal' and a word for 'White'. Quine says :

> Stimulus situations always differ, whether relevantly or not ; and just because volunteered responses come singly, the classes of situations under which the native happens to have volunteered [the words for 'Rabbit', 'Animal' and 'White'] are of course mutually exclusive, despite the hidden actual meanings of the words. How then is the linguist to perceive that the native would have

[2]L. Wittgenstein, *Preliminary Studies for the 'Philosophical Investigations', generally known as The Blue and Brown Books* (Oxford, 1958) : W. V. Quine, *Word and Object* (New York, 1960).

> been willing to assent to [the word for 'Animal'] in all the situations where he happened to volunteer [the word for 'Rabbit'], and in some but perhaps not all of the situations where he happened to volunteer [the word for 'White']? Only by taking the initiative and querying combinations of native sentences and stimulus situations . . . (*ibid.*)

The kind of logical structure Quine assumes to be built into the native language as between 'Rabbit', 'Animal' and 'White' is the kind that can be made explicit in terms of the connectives 'and' and 'or'; and so he seems to be saying that you cannot properly have even this degree of logical structure without also having negation. It is not entirely clear whether the impossibility is a theoretical one or merely a matter of a practical difficulty confronting the would-be translator. However, it can quite easily be argued, on Quine's own ground, that there need be no difficulty at all of the kind he envisages.

Let us suppose, for example, that our observer perches himself on a palm-tree with a notebook and just watches. Members of the tribe circulate below him and make noises. The range of noises is p, q, r, etc. Let us suppose that it is observed that noise p is sometimes made when feature X is present in the environment but never when feature X is absent; but that in the case of any other feature Y, Z, etc., that it occurs to the observer to test, noise p is made sometimes in the presence of that feature and sometimes in its absence. Similarly for noise q and feature Y and for noise r and feature Z, and so on.

We can also imagine that there is another kind of noise s which is sometimes made when features X and Y are both present but never when either is absent; or a noise t which is sometimes made in the presence of feature X and sometimes in the presence of Y but never when both are absent. And we can even imagine that there is a composite noise, consisting of noise K followed by two other separately meaningful noises, such that the composite noise is made sometimes when the features normally accompanying the other noises are both present but never when either is absent; and similarly a composite noise involving a noise interpretable as the operator 'A'. In the same way there may be longer locutions of manageable length, perhaps, say, with decreasing frequency according to length.

It might be reasonable, moreover, to allow some exceptions, and to spell out the 'sometimes' in some detail. Let us suppose, for example, that the interests of the tribe are exclusively zoological and that comments are regularly made on animals in the environment, with approximately equal frequencies for each animal in proportion to its incidence; or, if there are discrepancies, that they are generally explicable in terms of the animal's greater or less visibility or attention-catching propensity or interest. We assume, of course, that there are independent tests for these: the observer must be able to discover by observation what is "attention-catching", and so on. Similarly, if p is sometimes said in the absence of X this must be explicable as an error due to poor lighting or a slip of the tongue and occur more frequently when the native is distracted, intoxicated, etc.—for which there must also be independent tests.

Although one can envisage plenty of *possible* difficulties, this account reveals that there could be a case in which no difficulties arise; and this is all that is necessary to prove the point at issue. Simple induction from observation might give enough information to permit the observer to compile his translation-manual. We assume, of course, a certain amount of happy guesswork on the part of the observer; but even Quine, as we noticed, came down to this in the end.

Not all objections are answered by these considerations, which are essentially concerned with the possibility of providing the specimen language with a practically acceptable interpretation. A more theoretically-based objection should now be considered. It is admitted, it might be said, that an observer could or might accomplish all you say; but it does not follow from this that the native utterances are really *statements*. They would not in fact (it would be said) be statements in the absence of the possibility of denial or controversion: at best they would be some more primitive and undifferentiated kind of linguistic phenomenon.

Perhaps we should grant immediately that if a tribe were to evince no other kind of linguistic behaviour than that described then its utterances would not be statements in the full sense of the word. This concedes, however, very little: the point is largely verbal. What still needs to be emphasized is that controvertibility is only one among several of the criteria for distinguishing statements from locutions of other moods. The language described is lacking in one means of making this differentiation; but it may have others. In particular, there is one characteristic feature of statements that the native utterances may have; namely, that their function is to convey information. After a while even our observer may be confident that whenever he hears 'Gavagai' he may look and see a rabbit.

It is worth noticing that in the case of a mechanical communication link such as that between a satellite and its ground controller we speak sometimes of information passing, sometimes of interrogation, and sometimes of the transmission of orders or commands. But these categories are not grammatical ones. Their differentiation is in no way concerned with the presence or absence of a negation idiom in the transmitted signal.

Is it possible, in the absence of a negation idiom, for locutions to be "true" and "false"? Well yes: after a while the observer in the tree will begin to be able to make judgments such as 'He shouldn't have said 'Gavagai' then: there's no rabbit about at the moment'. But was this locution false in the full sense of the word? Shouldn't we describe it for preference as "inappropriate" or "infelicitous"? Since, in this language, it does not follow from the falsity of a given utterance that there is a related utterance that is true, the differentiation of falsity from infelicity is itself uncertain, and, if the question reduces to one of style, the simpler word is presumably to be preferred.

But how would the natives ever come to learn and use such a language if misuse could never be reproved ? It is not entirely clear why this question should matter ; but there is no reason to object to answering it. We need only suppose that in this particular tribe children have an instinctive drive towards nice linguistic conformity, coupled with a tendency to perform simple inductions. It is possible that human nature is not like this, but even this fact may be doubted ; for we have, in English, various moods of utterance besides the indicative, and our children seem to have no difficulty in picking up their rudiments in spite of the fact that adults never point out, because they never can, the *falsity* of a question, or command, or ejaculation.

We may finish with a theorem on the economy of the kind of language we have been discussing.

Let us assume that any given " feature " is present in the environment of our tribe only infrequently, though since there are many possible features there are usually a few present at any one time. This means only that they give names to deviations from the norm of their environment rather than to the norm itself, and that they do so on such a basis as to make it possible for them just to be able to keep track of all the describable events at any normal time.

And let us suppose that a reasonable proportion of the things there are to be said are in fact said, and that anything that can be said in fact is said reasonably often. These seem simply to be postulates to the effect that the language is moderately useful and economical. What is distinguished as a " feature " is something that is reasonably worthy, through rarity and interest, of distinction.

It follows immediately, from these postulates alone, that there is no negation in the language ; for a feature X and its negation cannot both be rare. There may, it is true, be locutions representing what we would sometimes call " negative features " : for example, if rabbits abounded in that part of the world, our rules would require that the tribe have no word for rabbit, but only a word for absence-of-rabbit, this being the more remarkable event. This, however, has nothing to do with denial, and we would have to say that " for them, the absence of rabbit is a positive event ".

It is perhaps unnecessary to add that implication would certainly be impossible ; for, given an operator 'C', we would immediately have the construction '$C\alpha\alpha$' as a possibility and, far from being rare, this would be a universally appropriate locution.

It follows that the language outlined is capable of greater economy than any logically more developed one, and that we can expect to find the logical structure of language limited in the way that of this language is limited whenever economy is important.

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